

---

**IMPROVEMENT OF SLR ACCURACY, A POSSIBLE NEW STEP**

**M. KASSER, ESGT, 18 Allée J. Rostand, BP 77, 91 002 EVRY France**

---

**Abstract**

The SLR technology experienced a large number of technical improvements since the early 1970<sup>ies</sup>, leading now to a millimetric instrumental accuracy. Presently it appears as useless to increase these instrumental performances as long as the atmospheric propagation delay suffers its actual imprecision. It has been proposed since many years to work in multiwavelength mode, but up to now the considerable technological difficulties of subpicosecond timing have seriously delayed such an approach.

Then a new possibility is proposed, using a device which is not optimized now for SLR but has already given good results in the lower troposphere for wind measurement: the association of a radar and a sodar. While waiting for the 2- $\lambda$  methodology, this one could provide an atmospheric propagation delay at the millimetre level during a few years with only little technological investment.

**I/ INTRODUCTION**

It has been pointed out since a long time that all space geodesy techniques have to deal with the same general problem, i. e. the crossing of the atmosphere to reach either an artificial satellite, or the moon, or a star, or a quasar, etc...

Nevertheless, if it is the same atmosphere for every techniques, the effect is known to be quite different for radiowave and for optical methodologies, the first ones suffering more than the other ones from the crossing of ionosphere (but this is corrected classically by two-frequencies methods) and from the troposphere transit (because of the atmospheric water vapor content which is quite unpredictable and has a strong impact on propagation of radio waves and not on optical ones). Considering these aspects, the general advantages of radio techniques compared with optical ones in geodesy are their all-weather capabilities, and their drawbacks are linked with their poor tropospheric correction quality.

As long as the instrumental accuracies were at a few centimeters level, this meteorological aspects were of secondary concern. This is no longer the case since many years with VLBI, who had to use costly water vapor radiometers to upgrade the tropospheric correction to an excellent level (and since this period, VLBI has got the best positioning precision among all space geodesy techniques). Since a few years too it is no longer the case with satellite laser ranging, which is able of an

internal instrumental precision of a few millimeters, to be compared with the final positioning accuracy of 2-3 cm. Of course the tropospheric corrections are not the only problems biasing the results (consider for example the poor world coverage of SLR stations), but significant improvements of this parameter must be researched. Many teams have pointed out that multiwavelength methodology could provide this required amelioration. Anyway it is quite clear that this will ask for a tremendous technological effort and will probably not be operational since some years. For that reason we have looked for a new solution, able to give better tropospheric correction with up-to-date techniques, even if probably not as efficient as 2-colour method, but immediately available.

## II/ THE ATMOSPHERIC CORRECTION FOR SLR

If  $n$  is the index of refraction,  $n = c_0 / c$  ( $c_0$  is the light celerity in vacuum and  $c$  in the atmosphere).

Classically we define the co-index of refraction as

$$N = (n - 1) \cdot 10^6$$

and for optical wavelengths, using for example Essen's formula:

$$N = A_\lambda (P_a / T) \cdot \{ 1 + (a - b \cdot t) \cdot P_a \} - B_\lambda (P_v / T)$$

where  $P_a$  is the atmospheric pressure,  $P_v$  is the water vapor pressure,  $T$  is the absolute temperature and  $t$  is the centigrade temperature,  $A_\lambda$  and  $B_\lambda$  are  $\lambda$ -dependant parameters, and  $a$  and  $b$  are constants. With a  $10^{-7}$  precision it is acceptable to use the simplified formulation:

$$N = A'_\lambda (P_a / T) - B_\lambda (P_v / T)$$

We define also the geometric length of the optical ray  $L_g$ , the optical path  $L_o$ , and the geometric distance  $L$  between two points  $s_0$  and  $s_1$  being one at the ground level and the other beyond the atmosphere, at the satellite level:

$$L_g = \int_{s_0}^{s_1} ds \qquad L_o = \int_{s_0}^{s_1} n(s) \cdot ds$$

If we call  $\Delta L = L_o - L$ , it is a function of the angle  $\alpha$  between the vertical and the direction of the satellite. We notice that:

$$\Delta L = (L_o - L_g) + (L_g - L)$$

As the curvature of the ray path is low, and as this curvature is experienced only on a small range (a few tens of kilometers), the difference  $L_g - L$  is generally considered as negligible. On another hand,  $\Delta L$  can be expressed as:

$$\Delta L = f(\alpha) \cdot \Delta L_{\text{vertical}}$$

And the function  $f(\alpha)$  may be found in (Berrada-Baby *et al.*, 1987 or Akhundov et Stoskil, 1992). We have now to deal with  $\Delta L$  for  $\alpha = 0$ .  $h_0$  being the elevation of the SLR station above "sea level",

$$\Delta L = \int_{h_0}^{\infty} N(h) \cdot dh$$

Considering the very low dependance of the result regarding the water vapor for these optical wavelengths, we will focus on the "dry" part of this expression  $\Delta L_d$  :

$$\Delta L_d = \int_{h_0}^{\infty} A \frac{P_a}{T} \cdot dh$$

And,  $\rho(h)$  being the air density,  $T(h)$  the absolute temperature,  $P(h)$  the atmospheric pressure and  $g(h)$  the acceleration of the pesanteur at the elevation  $h$ , with  $R = 287 \text{ J.kg/}^\circ\text{K}$

$$* P_a = \rho(h) \cdot R \cdot T(h)$$

$$* d P(h) = \rho(h) \cdot g(h) \cdot dh$$

so that,

$$\Delta L_d = \int_{h_0}^{\infty} \frac{dP(h)}{g(h)} \cdot A \cdot R$$

At this level we may have different assumptions:

1<sup>st</sup> hyp.  $g(h) = g_0 = \text{constant}$ . Then  $\Delta L_d = A \cdot R \cdot P_0 / g_0$

2<sup>nd</sup> hyp.  $g(h) = g_0 / (1 + 2 \cdot h / r_0)$  closer to the reality. We find then:

$$\Delta L_d = \frac{A \cdot R \cdot P_0}{g_0} \left( 1 + \frac{2 R \beta}{r_0 \cdot \frac{\beta}{T_0} (g_0 + R \cdot \beta) - 2 \cdot g_0} \right)$$

( $\beta$  is the gradient of  $T$ ). For  $P_0 = 1000 \text{ mbar}$ , we find  $\Delta L_d \approx 2,3 \text{ meters}$

And the difference between these two models is below 1 cm : various studies show (Berrada-Baby, 1987) that this discrepancy is around 5 mm, and is quite stable and thus easy to model. On another hand, the differences between the second model and real data deduced from (Cira 1965) campaigns are quite small (1 mm typically).

So we conclude that it is extremely important to measure  $P_0$  with an excellent precision (1 mbar of error induces 2.3 mm on the atmospheric correction). And one must take into account the fact that the function  $f(\alpha)$  (that is the air-mass number) will multiply these values by numbers up to 2.5 in operational SLR measurements.

If the temperature is measured in the first 5 kilometers, looking at fig. 1 and fig. 2, we observe that half the total correction is already acquired (fig. 1), and that the upper layers are quite well defined by the profile in the troposphere. If we compare these facts with the abovementioned value of 5 mm, whose noise is around 1-2 mm, multiplied by the air-mass value (1 to 2.5), i. e. less than 5 mm, it is clear that any temperature profile of good precision acquired in the troposphere will leave a residual error on the atmospheric correction at the millimeter level.

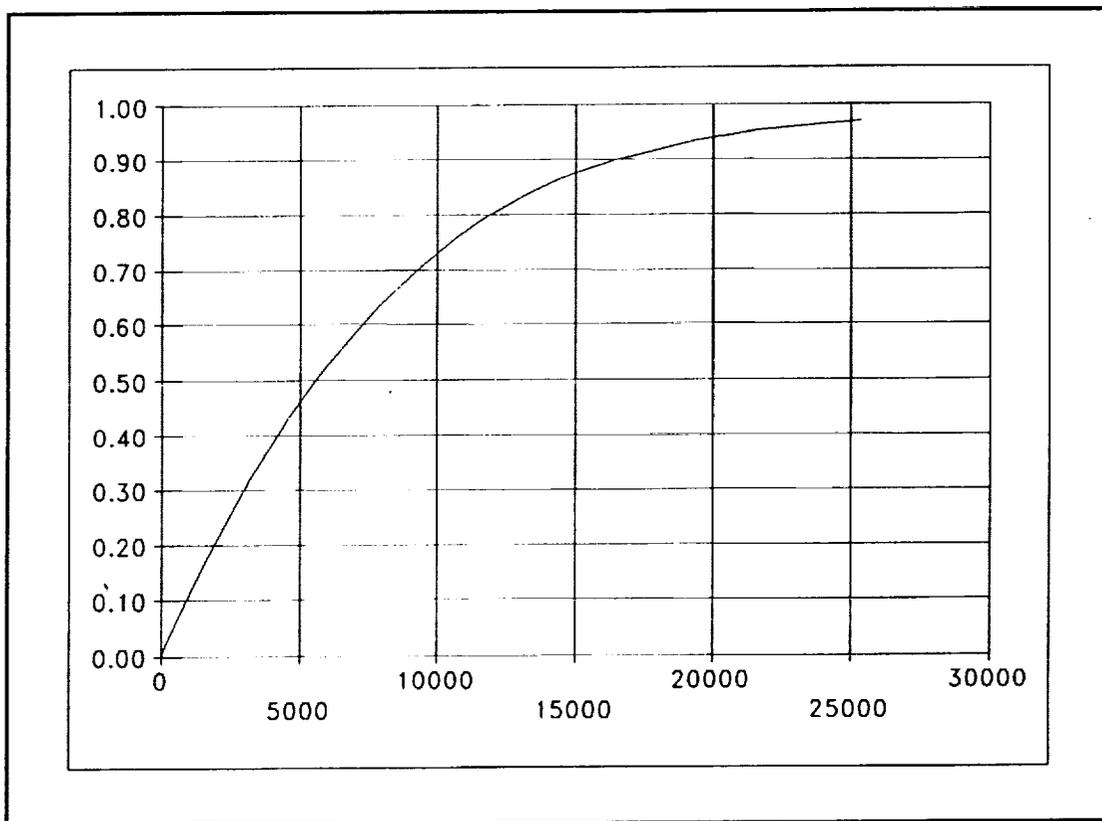


Fig. 1: Fraction (Y-axis) of the total atmospheric correction for a vertical transit of a laser pulse, from the ground level up to a given altitude (X-axis, in meters). Computed from an observed radio-profile in Greece, 1990.

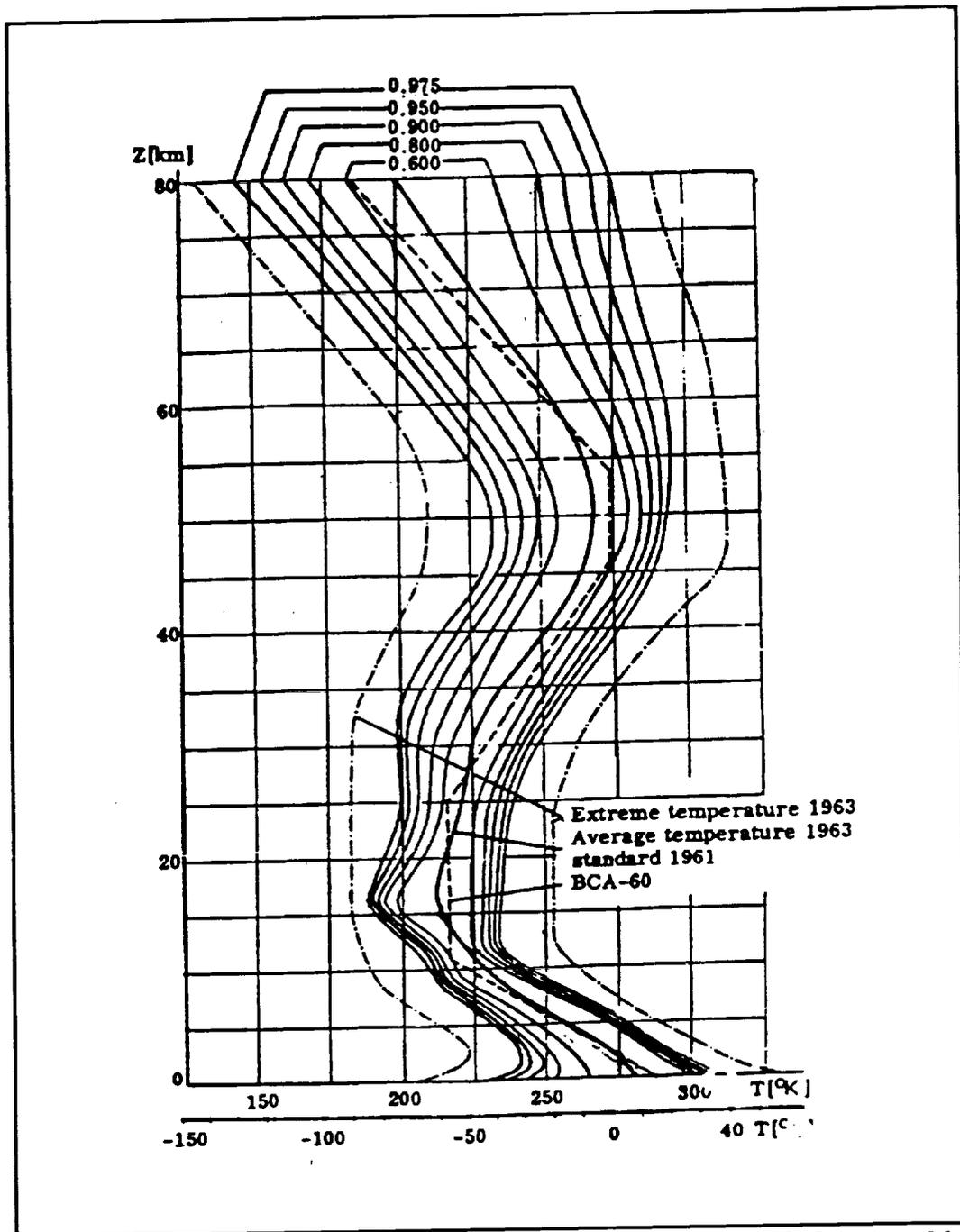


Fig. 2: Typical temperature profiles (from CIRA 1965). If the possible variations are quite large, it is also noticeable that the profiles have the same topological aspects and are quite "parallel", with no intersection from one to another.

### III/ PROPOSITIONS

The first one is obviously to measure  $P_0$  quite carefully. It is necessary to calibrate often the barometers employed, to measure preferably the pressure close to the level of the axes intersection of the telescope (it is generally the case). *It must be possible to measure the pressure with an absolute precision < 0.2 mbar.*

Considering the efficiency of barometric levelling, whose precision may reach one meter, we observe that it means that constant pressure surfaces are quite horizontal, so that it is useless to measure the horizontal gradient of  $P_0$  in order to correct its effect in the direction of the sight, provided the weather is reasonably quiet.

Anyway, it seems mostly advisable to improve the correction by a good measurement of  $T(h)$  in the direction of the satellite, at the  $0.1^\circ\text{C}$  level, up to an elevation of 3 to 10 km.

*In these conditions, one may be sure to get an atmospheric correction better than 1 mm.* It is easy to notice that such a precision with 2-wavelength SLR will require a 20 times better precision on the differential time-of-flight measurement between the two colors (i. e. 0.5 picosecond), which is quite uneasy to reach.

The solution we propose for an easy measurement of the temperature profile along the line of sight is a SODAR (Acoustic LIDAR) associated with a RADAR, the acoustic and the radio wave having the same wavelength. The radar is used to track the acoustic wave, so it allows to measure the speed of the sound in the atmosphere, which in turn provides an excellent temperature profile (at the  $0.1^\circ\text{C}$  level). Such instruments (e. g. Remtech) are now used close to airports, in order to measure winds and wind shears, and this technology is already available. The common ranges are 3 to 5 km, but large instruments may reach 10 km.

#### IV/ CONCLUSIONS

We propose that an experiment be carried with a radar/sodar equipment close to an up-to-date SLR station in order to check the possible improvements that such data could provide to the laser data. This solution is probably an excellent alternative allowing to wait for the 2-wavelength generation of SLR stations.

On another hand, we insist on the necessity to check quite carefully the barometric equipments used in SLR stations, as the  $P_s$  parameter is by far the most important to perform a good atmospheric correction.

#### BIBLIOGRAPHY

H. BERRADA-BABY, P. GOLE & J. LAVERGNAT

*Effets de la troposphere sur les mesures de distance Terre-Satellite.  
Application au projet DORIS.*

CRPE/158 Technical note. June 1987

RPE/ETP, 38-40 rue du Général LECLERC, 92 131 ISSY-LES-MOULINEAUX,  
France

CIRA 1965

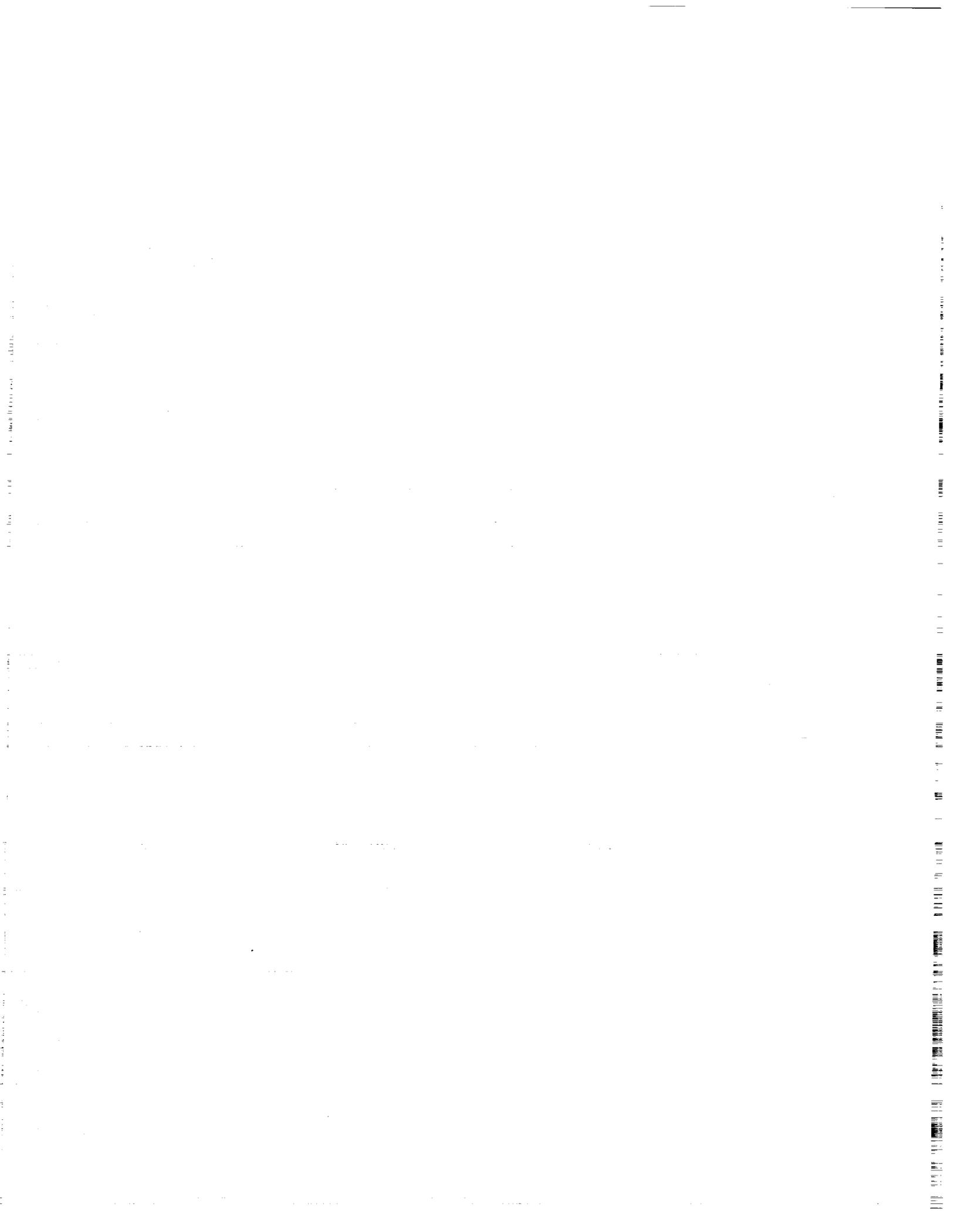
North-Holland Publishing Company, Amsterdam Edition, 1965

T. A. AKHUNDOV, A. A. STOTSKII

*Zenith angle dependance of path length through the troposphere*

Proceedings of the symposium "Refraction of transatmospheric signals in geodesy".  
Den Haag, 19-22 May 1992, pp 37-41

REMTECH Company, technical papers on acoustic wind profilers.  
Remtech Inc., 1040 Townley Circle, Longmont, CO 80 501, USA



# **Operational Software Developments**

